

How the single supply chain works

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Two sectors, each of them has a monopolist. The first sector is a farm that produces beef, the second sector is a restaurant that turns beef into food. The farm requires no input. It takes one unit of beef to make one unit of food.

The restaurant sells to the demand function (representing the end consumers)

$$q_F = 101 - p_F$$

Each firm faces its independent labor supply function; its daily wage curve is

$$w = \frac{5}{7}q$$

The farm maximizes:

$$\Pi_B = p_B q_B - q_B \frac{5}{7} q_B$$

Derive to find the optimum:

$$p_B = \frac{10}{7} q_B$$

The restaurant maximizes:

$$\Pi_F = (101 - q_F)q_F - q_F \left(\frac{5}{7}\right)q_F - q_F p_B$$

Derive to find the optimum:

$$q_F = (101 - p_B) \frac{7}{24}$$

Now find where the best responses intersect:

$$\begin{cases} p_B = \frac{10}{7}q_B \\ q_F = (101 - p_B)\frac{7}{24} \\ q_F = q_B \end{cases}$$

We find that the solutions are

$$q = 20.794, p = 29.706$$