Simple Macro

Ernesto

June 2, 2014

1 Simple world

There are N people in the world. Each day a person has to decide whether to be farmer or worker. As worker, a person gets daily wage w and produces a fixed number γ of manufactured goods. As farmer, the person produces η units of agricultural goods every day. η is heterogeneous: the first farmer was born with $\eta = 1$, the second with $\eta = 2$, and the last $\eta = N$. Personal agricultural production η is not a function of how many other people farm, it is a personal parameter of each agent.

In aggregate, L_m is the number of people who works in manufacturing, L_a in farming.

$$L_a + L_m = N$$

Given L_m workers, total daily production is:

$$Q_m = \gamma L_m$$

A farmer will become a worker only if wages are above or equal its personal η. Because I expect the "worst" farmers (those with the lowest η) to become workers first, daily wages are:

$$w_m \geq L_m$$

The L_a people with the highest ηs will be farmers, total agricultural production is:

$$q_a = \frac{N(N+1)}{2} - \frac{L_m(L_m+1)}{2}$$

(that's the sum of all integers between $L_m + 1$ and N)which simplifies in:

$$q_a = \frac{L_a(N+1) - L_a L_m}{2}$$

Every person has the same utility function:

$$U = (q_a + 1)^{.5}(q_m + 1)^{.5}$$

setting MRS equal to price ratio, and remembering that a is the monetary commodity so its price is one:

$$\begin{aligned}
\text{MRS} &= \frac{p_m}{p_a} \\
\frac{\frac{\partial U}{\partial m}}{\frac{\partial U}{\partial a}} &= p_m \\
q_a &= p_m(q_m + 1) - 1
\end{aligned}$$

For workers, the budget constraint is:

$$q_a + p_m q_m = L_m$$

Where L_m is the wage.

$$p_m(q_m + 1) - 1 + p_m q_m = L_m$$

$$q_m = \frac{L_m - p_m + 1}{2p_m}$$

A farmer budget constraint is contingent on its own personal production:

$$q_a + p_m q_m = \eta_i$$

$$p_m(q_m + 1) - 1 + p_m q_m = \eta_i$$

$$q_m = \frac{\eta_i - p_m + 1}{2p_m}$$

Let's aggregate demands. We sum up the demand for all workers, which are all identical and the demand for farmers which are heterogeneous:

$$Q_m = L_m \times \text{Worker demand}) + \sum \text{Farmer Demand}$$

$$Q_m = L_m \left(\frac{L_m}{2p_m} + \frac{1 - p_m}{2p_m} \right) + \sum \left(\frac{\eta}{2p_m} + \frac{1 - p_m}{2p_m} \right)$$

Now, $\sum \eta_i$ is just total agricultural production Q_a.

$$Q_m = \frac{L_m^2}{2p_m} + L_m \frac{1 - p_m}{2p_m} + \frac{Q_a}{2p_m} + L_a \frac{1 - p_m}{2p_m}$$

$L_a + L_m$ is everyone so:

$$Q_m = \frac{L_m^2}{2p_m} + \frac{Q_a}{2p_m} + N \frac{1 - p_m}{2p_m}$$

$$Q_m = \frac{L_m^2 + Q_a + N(1 - p_m)}{2p_m}$$

Or if we want the price:

$$p_m = \frac{q_a + L_m^2 + N}{2q_m + N}$$

Notice that L_m is really a function of q so we could simply further, but for now keep it like that.

2 Zero profits

Manufacturing must make zero profits

$$p_m q_m - wL_m = 0$$

$$p_m \frac{L_m^2 + Q_a + N(1-p_m)}{2p_m} - L_m^2 = 0$$

Now if we add this condition, we have a sistem of 5 equations in 5 unknowns (after setting $N = 50$, $\gamma = 10$):

$$\begin{cases} p_m \frac{L_m^2 + Q_a + 50(1-p_m)}{2p_m} - L_m^2 = 0 & \text{Profits are 0} \\ Q_a = \frac{50(50+1)}{2} - \frac{L_m(L_m+1)}{2} & \text{Agricultural production definition} \\ L_a + L_m = N & \text{Everybody is employed} \\ Q_m = 10L_m & \text{Linear manufacturing production} \\ p_m = \frac{Q_a + L_m^2 + 50}{2Q_m + 50} & \text{Demand function} \end{cases}$$

And you end up with

$$\begin{aligned} L_m &= 27.9441 \approx 28 \\ L_a &\approx 22 \\ Q_m &\approx 280 \\ Q_a &\approx 869 \\ p &\approx 2.70 \end{aligned}$$

If $N = 200$ instead:

$$\begin{aligned} L_m &= 109.7 \approx 110 \\ L_a &\approx 90 \\ Q_m &\approx 1100 \\ Q_a &\approx 13995 \\ p &\approx 10.95 \end{aligned}$$