```
with(geometry);
```
[Apollonius, AreCollinear, AreConcurrent, AreConcyclic, AreConjugate, AreHarmonic, AreOrthogonal, AreParallel, ArePerpendicular, AreSimilar, AreTangent, CircleOfSimilitude, CrossProduct, CrossRatio, DefinedAs, Equation, EulerCircle, EulerLine, ExteriorAngle, ExternalBisector, FindAngle, GergonnePoint, GlideReflection, HorizontalCoord, HorizontalName, InteriorAngle, IsEquilateral, IsOnCircle, IsOnLine, IsRightTriangle, MajorAxis, MakeSquare, MinorAxis, NagelPoint, OnSegment, ParallelLine, PedalTriangle, PerpenBisector, PerpendicularLine, Polar, Pole, RadicalAxis, RadicalCenter, RegularPolygon, RegularStarPolygon, SensedMagnitude, SimsonLine, SpiralRotation, StretchReflection, StretchRotation, TangentLine, VerticalCoord, VerticalName, altitude, apothem, area, asymptotes, bisector, center, centroid, circle, circumcircle, conic, convexhull, coordinates, detail, diagonal, diameter, dilatation, directrix, distance, draw, dsegment, ellipse, excircle, expansion, foci, focus, form, homology, homothety, hyperbola, incircle, inradius, intersection, inversion, line, medial, median, method, midpoint, orthocenter, parabola, perimeter, point, powerpc, projection, radius, randpoint, reciprocation, reflection, rotation, segment, sides, similitude, slope, square, stretch, tangentpc, translation, triangle, vertex, vertices] **(1)**

```
pointA := point(A, x_A, y_A);
```
$$A \qquad (2)$$

```
pointB := point(B, x_B, y_B);
```
$$B \qquad (3)$$

```
pointC := point(C, x_C, y_C);
```
$$C \qquad (4)$$

```
unassign('cond'); AreCollinear(A, B, C, cond);
AreCollinear: hint: could not determine if x_A*y_B-x_A*y_C-x_B*
y_A+x_B*y_C+x_C*y_A-x_C*y_B is zero
```
$$FAIL \qquad (5)$$

```
op(1, cond);
```
$$x\_A\, y\_B - x\_A\, y\_C - x\_B\, y\_A + x\_B\, y\_C + x\_C\, y\_A - x\_C\, y\_B \qquad (6)$$

```
assume(0 < op(1, cond));
triangle(T, [A, B, C]);
```
$$T \qquad (7)$$

```
circumcircle(CC, T,'centername'=DD);
```
$$CC \qquad (8)$$

```
detail(DD);
```

name of the object	DD
form of the object	$point2d$
coordinates of the point	$\left[\frac{x\_A\sim^2\, y\_B\sim - x\_A\sim^2\, y\_C\sim - x\_B\sim^2\, y\_A\sim + x\_B\sim^2\, y\_C\sim + x\_C\sim^2\, y\_A\sim - x\_C\sim^2\, y\_B\sim}{2\,(x\_A\sim\, y\_B\sim - x\_A\sim\, y\_C\sim - x\_B\sim\, y\_A\sim} \right.$

```
detail(CC);
assume that the names of the horizontal and vertical axes are _x
```

`and _y, respectively`

name of the object CC

form of the object $circle2d$

name of the center DD

coordinates of the center $\left[\frac{x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A{\sim} - x\_C{\sim}^2\, y\_B{\sim}}{2\,(x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A}\right.$

radius of the circle $\sqrt{\left(x\_A{\sim} - \frac{x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A{\sim} -}{2\,(x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim}}\right.}$

equation of the circle $\_x^2 + \_y^2 - \frac{\_x\,(x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A}{x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C}$

$E4 := Equation(CC);$ *# Note that you need to enter x_D and y_D as the coordinates.*

`` `while processing result` ``

$$
\begin{aligned}
& x\_D^2 + y\_D^2 - \big(x\_D\,(x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A{\sim} \\
&\quad - x\_C{\sim}^2\, y\_B{\sim} + y\_A{\sim}^2\, y\_B{\sim} - y\_A{\sim}^2\, y\_C{\sim} - y\_A{\sim}\, y\_B{\sim}^2 + y\_A{\sim}\, y\_C{\sim}^2 + y\_B{\sim}^2\, y\_C{\sim} \\
&\quad - y\_B{\sim}\, y\_C{\sim}^2)\big) \big/ (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A{\sim} + x\_B{\sim}\, y\_C{\sim} + x\_C{\sim}\, y\_A{\sim} \\
&\quad - x\_C{\sim}\, y\_B{\sim}) + \big(y\_D\,\big(\text{-}x\_A{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2) + x\_A{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) \\
&\quad + x\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) + y\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) - x\_B{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) \\
&\quad + x\_C{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2)\big)\big) \big/ (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A{\sim} + x\_B{\sim}\, y\_C{\sim} \\
&\quad + x\_C{\sim}\, y\_A{\sim} - x\_C{\sim}\, y\_B{\sim}) + \frac{1}{4}\,(x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} \\
&\quad + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A{\sim} - x\_C{\sim}^2\, y\_B{\sim} + y\_A{\sim}^2\, y\_B{\sim} - y\_A{\sim}^2\, y\_C{\sim} - y\_A{\sim}\, y\_B{\sim}^2 \\
&\quad + y\_A{\sim}\, y\_C{\sim}^2 + y\_B{\sim}^2\, y\_C{\sim} - y\_B{\sim}\, y\_C{\sim}^2)^2 \big/ (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A{\sim} \\
&\quad + x\_B{\sim}\, y\_C{\sim} + x\_C{\sim}\, y\_A{\sim} - x\_C{\sim}\, y\_B{\sim})^2 + \frac{1}{4}\,\big(\text{-}x\_A{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2) \\
&\quad + x\_A{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) + x\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) + y\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) \\
&\quad - x\_B{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) + x\_C{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2)\big)^2 \big/ (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} \\
&\quad - x\_B{\sim}\, y\_A{\sim} + x\_B{\sim}\, y\_C{\sim} + x\_C{\sim}\, y\_A{\sim} - x\_C{\sim}\, y\_B{\sim})^2 - \Big(x\_A{\sim} \\
&\quad - \frac{1}{2}\,(x\_A{\sim}^2\, y\_B{\sim} - x\_A{\sim}^2\, y\_C{\sim} - x\_B{\sim}^2\, y\_A{\sim} + x\_B{\sim}^2\, y\_C{\sim} + x\_C{\sim}^2\, y\_A{\sim} - x\_C{\sim}^2\, y\_B{\sim} \\
&\quad + y\_A{\sim}^2\, y\_B{\sim} - y\_A{\sim}^2\, y\_C{\sim} - y\_A{\sim}\, y\_B{\sim}^2 + y\_A{\sim}\, y\_C{\sim}^2 + y\_B{\sim}^2\, y\_C{\sim} - y\_B{\sim}\, y\_C{\sim}^2) \big/ \\
&\quad (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A{\sim} + x\_B{\sim}\, y\_C{\sim} + x\_C{\sim}\, y\_A{\sim} - x\_C{\sim}\, y\_B{\sim})\Big)^2 \\
&\quad - \Big(y\_A{\sim} + \frac{1}{2}\,\big(\text{-}x\_A{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2) + x\_A{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) \\
&\quad + x\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) + y\_A{\sim}^2\,(x\_B{\sim} - x\_C{\sim}) - x\_B{\sim}\,(x\_C{\sim}^2 + y\_C{\sim}^2) \\
&\quad + x\_C{\sim}\,(x\_B{\sim}^2 + y\_B{\sim}^2)\big) \big/ (x\_A{\sim}\, y\_B{\sim} - x\_A{\sim}\, y\_C{\sim} - x\_B{\sim}\, y\_A{\sim} + x\_B{\sim}\, y\_C{\sim}
\end{aligned}
\qquad \textbf{(11)}
$$

$$+x\_C{\sim}\,y\_A{\sim}-x\_C{\sim}\,y\_B{\sim})\Big)^2=0$$

 # That was equation 4 of the paper

normal(*E4* − *eval*(*E4*, {x_D = x_D + c, y_D = y_D + c, x_A = x_A + c, y_A = y_A + c, x_B = x_B + c, y_B = y_B + c, x_C = x_C + c, y_C = y_C + c}));

$$0=0 \tag{12}$$

 # That's the verification

E5 := *eval*(*E4*, y_A = 0);   *# equation 5 of the paper*

$$\begin{aligned}
&x_D^2+y_D^2\\
&-\left(x_D\left(x_A{\sim}^2\,y_B{\sim}-x_A{\sim}^2\,y_C{\sim}+x_B{\sim}^2\,y_C{\sim}-x_C{\sim}^2\,y_B{\sim}+y_B{\sim}^2\,y_C{\sim}\right.\right.\\
&\left.\left.-y_B{\sim}\,y_C{\sim}^2\right)\right)\Big/\left(x_A{\sim}\,y_B{\sim}-x_A{\sim}\,y_C{\sim}+x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}\right)+\left(y_D\left(\right.\right.\\
&-x_A{\sim}\left(x_B{\sim}^2+y_B{\sim}^2\right)+x_A{\sim}\left(x_C{\sim}^2+y_C{\sim}^2\right)+x_A{\sim}^2\left(x_B{\sim}-x_C{\sim}\right)\\
&\left.\left.-x_B{\sim}\left(x_C{\sim}^2+y_C{\sim}^2\right)+x_C{\sim}\left(x_B{\sim}^2+y_B{\sim}^2\right)\right)\right)\Big/\left(x_A{\sim}\,y_B{\sim}-x_A{\sim}\,y_C{\sim}\right.\\
&\left.+x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}\right)\\
&+\frac{1}{4}\left(x_A{\sim}^2\,y_B{\sim}-x_A{\sim}^2\,y_C{\sim}+x_B{\sim}^2\,y_C{\sim}-x_C{\sim}^2\,y_B{\sim}+y_B{\sim}^2\,y_C{\sim}\right.\\
&\left.-y_B{\sim}\,y_C{\sim}^2\right)^2\Big/\left(x_A{\sim}\,y_B{\sim}-x_A{\sim}\,y_C{\sim}+x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}\right)^2-\Big(x_A{\sim}\\
&-\frac{1}{2}\left(x_A{\sim}^2\,y_B{\sim}-x_A{\sim}^2\,y_C{\sim}+x_B{\sim}^2\,y_C{\sim}-x_C{\sim}^2\,y_B{\sim}+y_B{\sim}^2\,y_C{\sim}\right.\\
&\left.-y_B{\sim}\,y_C{\sim}^2\right)\Big/\left(x_A{\sim}\,y_B{\sim}-x_A{\sim}\,y_C{\sim}+x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}\right)\Big)^2=0
\end{aligned} \tag{13}$$

normal(*E5* − *eval*(*E5*, {x_D = x_D + c, x_A = x_A + c, x_B = x_B + c, x_C = x_C + c}));

$$0=0 \tag{14}$$

 # second verification

E6 := *eval*(*E5*, x_A = 0); *# equation 6*

$$\begin{aligned}
&x_D^2+y_D^2-\frac{x_D\left(x_B{\sim}^2\,y_C{\sim}-x_C{\sim}^2\,y_B{\sim}+y_B{\sim}^2\,y_C{\sim}-y_B{\sim}\,y_C{\sim}^2\right)}{x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}}\\
&+\frac{y_D\left(-x_B{\sim}\left(x_C{\sim}^2+y_C{\sim}^2\right)+x_C{\sim}\left(x_B{\sim}^2+y_B{\sim}^2\right)\right)}{x_B{\sim}\,y_C{\sim}-x_C{\sim}\,y_B{\sim}}=0
\end{aligned} \tag{15}$$

E7 := *eval*(*E4*, {y_A = 0, y_B = 0, x_B = -x_A});

 # equation 7 except that the x_A isn't cancelled in the fraction, and the numerator isn't simplified

$$\begin{aligned}
&x_D^2+y_D^2\\
&-\frac{1}{2}\,\frac{1}{x_A{\sim}\,y_C{\sim}}\left(y_D\left(-x_A{\sim}^3+2\,x_A{\sim}\left(x_C{\sim}^2+y_C{\sim}^2\right)+x_A{\sim}^2\left(-x_A{\sim}\right.\right.\right.\\
&\left.\left.\left.-x_C{\sim}\right)+x_C{\sim}\,x_A{\sim}^2\right)\right)-x_A{\sim}^2=0
\end{aligned} \tag{16}$$