Efficient Solvers for Atmospheric Tomography

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joint work with Ronny Ramlau (JKU Linz) & Roberto Biasi (Microgate)

Reduced Order Modelling, Simulation and Optimization of Coupled Systems (ROMSOC)

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Outline

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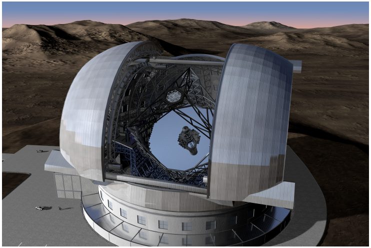
Extremely Large Telescope (ELT)

- European Southern Observatory (ESO) located in Garching, Munich
- construction and operation of astronomical telescopes

  - built in the Atacama desert, Chile
  - primary mirror: 39 m

  - 2005: first plans for ELT
  - 2008: Austria joins ESO
  - 2014: decision to build ELT
  - 2025: first light

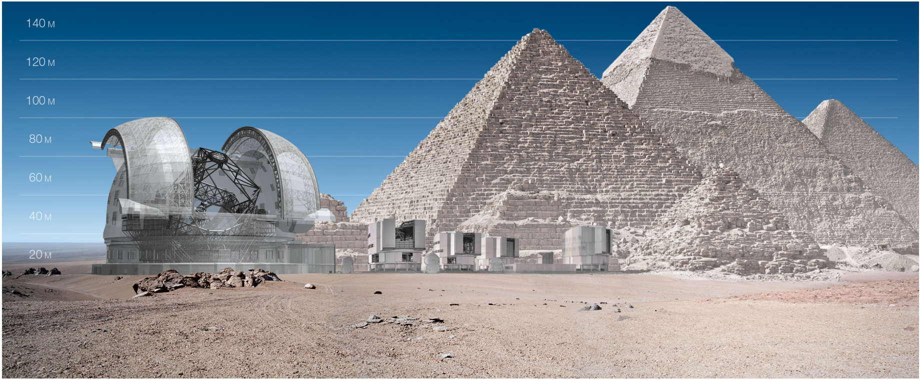


Credit: ESO

- Austrian scientific contribution:

  Mathematical Algorithms and Software for ELT Adaptive Optics

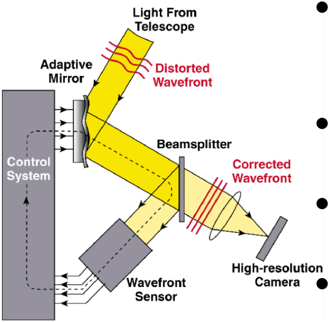
The world's biggest eye on the sky



ELT vs. the Pyramids (Credit: ESO)

Adaptive Optics (AO)

Technique that removes aberrations during the imaging process through deformable mirrors.

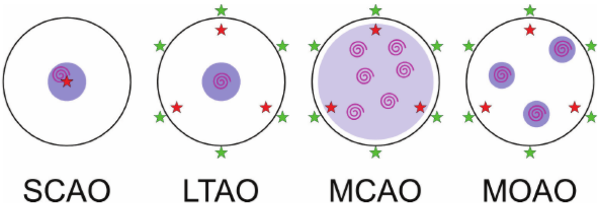


- light detected from reference sources (NGS, LGS) close to the astronomical object of interest
- WFS measure aberrations of the light caused by atmospheric perturbations
- **task of AO system:** remove distortions to allow a sharper view
- adaptive component - deformable mirror
- correction in real-time, i.e., at around 500-1000 Hertz

Credit: Claire Max, UCSC

Operating modes

- basic operating system: **SCAO** (Single Conjugate Adaptive Optics), 1 light source, applied if a NGS is sufficiently near to the observed object
- complex operating systems: good correction in several directions or over wide field of view, several guide stars
  - **MCAO** (Multi Conjugate AO)
  - **LTAO** (Laser Tomography AO)
  - **MOAO** (Multi Object AO)



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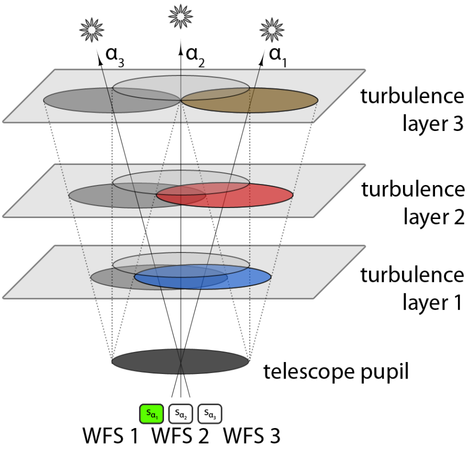
Atmospheric Tomography



- assuming layered model of the atmosphere
- good correction in several directions or over a wide field
- using several guide stars
- **goal:** reconstruct turbulent layers from WFS measurements
- mathematical problem formulation:

$$s = A\phi$$

$\Longrightarrow$ **ill-posed inverse problem,** requires **regularization**

The tomography problem - Bayesian framework

$$s = A\phi + \eta$$

Advantage: allows to incorporate statistics of turbulence and noise

Maximum a-posteriori estimate (**MAP**)

$$(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s,$$

where C_ϕ and C_η are the covariance matrices of layers ϕ and noise η.

Challenges:

- inverse problem
- computationally very expensive operations
- to be solved in real-time

$\Rightarrow$ **need an efficient solver**

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Standard Approach: Matrix Vector Multiplication (MVM)

We want to solve:

$$(A^T C_\eta^{-1} A + C_\phi^{-1})\phi = A^T C_\eta^{-1} s$$

Soft Real Time (SRT):

1. Compute the inverse of the tomography operator:

$$R := (A^T C_\eta^{-1} A + C_\phi^{-1})^{-1} A^T C_\eta^{-1}$$

2. Multiply R by projection matrix P to obtain DM commands u from layers ϕ

Hard Real Time (HRT):

Apply matrix vector multiplication to get DM commands:

$$u = PR \cdot s$$

Finite Element Wavelet Hybrid Algorithm (FEWHA)

We want to solve:

$$(A^T C_\eta^{-1} A + C_\phi^{-1})\phi = A^T C_\eta^{-1} s$$

Use clever transformations - dual domain discretization:

$$(\boldsymbol{W}^{-T} \hat{A}^T C_\eta^{-1} \hat{A} \boldsymbol{W}^{-1} + \alpha D)c = \boldsymbol{W}^{-T} \hat{A}^T C_\eta^{-1} s$$

Hard Real Time:

1. Compute right hand side: $b^n = \boldsymbol{W}^{-T} \hat{A}^T C_\eta^{-1} s$
2. Update residual: $r^n = (b^n - b^{n-1}) + r^{n-1}$
3. Perform atmospheric tomography with an iterative approach: Preconditioned Conjugate Gradient Method $(c^n, r^n) = PCG(c^{n-1}, r^{n-1})$
4. Projection: $u = Q\boldsymbol{W}^{-1} c^n$

[1] *M. Yudytskiy, T. Helin, R. Ramlau, A finite element - wavelet hybrid algorithm for atmospheric tomography*

Dual Domain Discretization

Turbulence statistics in the wavelet domain:

- lead to a diagonal approximation of C_ϕ
- Daubechies 3 wavelets
- sparse but no numerical efficient representation $\tilde{A}$ of A

Atmospheric tomography operator in finite element basis:

- continuous piecewise bilinear functions for wavefronts and layers
- obtain a sparse representation $\hat{A}$ of A

Mapping between finite element and wavelet domains:

$$\boldsymbol{W} := diag(\delta_1 W, ..., \delta_L W)$$

Finite Element Wavelet Hybrid Algorithm

We want to solve:

$$(A^T C_\eta^{-1} A + C_\phi^{-1})\phi = A^T C_\eta^{-1} s$$

Use clever transformations - dual domain discretization:

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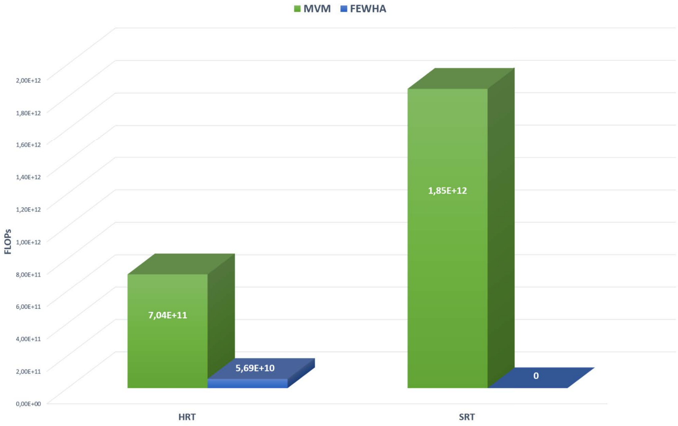
Test Setting - MAORY

- first light instruments for the ELT
- designed to compensate for distortions caused by atmospheric turbulences
- using atmospheric tomography with MCAO
- collaboration with Microgate (responsible for RTC design)

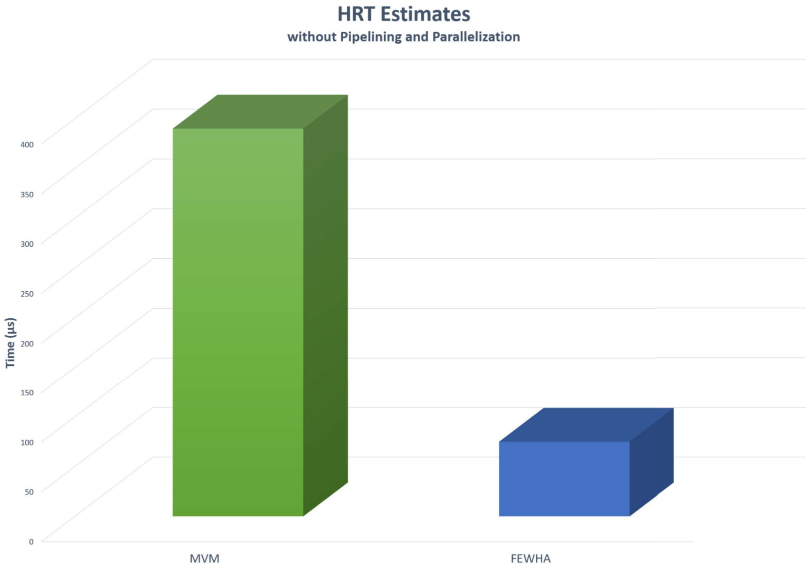
Mirrors:	M4, M5 and DM1, DM2
Guide Stars:	6 LGS, 3 NGS
WFS:	12 WFS
Number of WFS-HO slopes:	9600
Number of DOFs:	6462
Max frequency WFS-HO:	500 Hz
PR matrix update:	360 s
Size of PR matrix:	6462 x 9600

Floating Point Operations (FLOPs)



⇒ FEWHA **12 times faster** than the MVM for hard real time
⇒ FEWHA has **no additional soft real time** computational load

Estimated Results on NVIDIA Tesla V100



Memory Usage

Assumption: single precision (32 bit) floating point numbers

	MVM	FEWHA
Hard Real Time Memory Usage	3GB	15MB
Soft Real Time Memory Usage	50GB	-
	53GB	**15MB**

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Conclusion

MVM:

- very slow for ELT-sized problems
- good parallelizable and pipelinable
- easy to implement on GPUs (common libraries available)

FEWHA:

- really fast algorithm for ELT-sized problems
- based on iterative method
- not pipelinable
- high level of data exchange
- hard to parallelize and implement on GPU

Outline

Ongoing and Future Work

Compare algorithms on RTC hardware:
⇒ parallel implementation of FEWHA on a Tesla V100
⇒ comparison of the two algorithms with parallelization and pipelining

Challenge - bring FEWHA onto the GPU:

- **maximize bandwidth**
  use as much fast memory and as little slow-access memory as possible
- **minimize thread divergence**
  threads follow different execution paths
- **maximize hardware utilization**
  optimize number of used threads and blocks

Thank you!

[1] *M. Yudytskiy, T. Helin, R. Ramlau, A finite element - wavelet hybrid algorithm for atmospheric tomography.*

[2] *C. Patauner, R. Biasi, M. Andrighettoni, G. Angerer, D. Pescoller, F. Porta, D. Gratadour, FPGA based microserver for high performance real-time computing in Adaptive Optics*, AO4ELT5, 2017.

[3] *I. Foppiano, E. Diolaiti, P. Ciliegi, MAORY Adaptive Optics Real-Time Computer User Requirements.*

[4] *Doucet N. , Gratadour D., Ltaief H., Efficient Supervision Strategy for Tomographic AO Systems on E-ELT.*

[5] *CUDA C best practice guide.*

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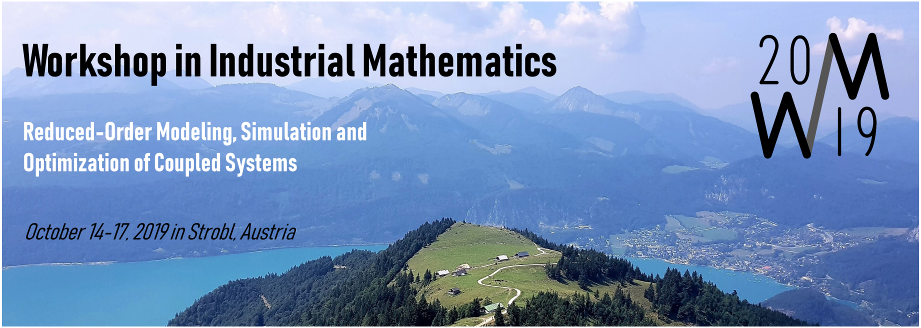

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